Units in Binary Black Hole Simulations

I. INTRODUCTION

In a binary black hole system, the independent units are those of mass, length and time, and all others such as energy, momentum etc can be written in terms of these. Suppose we choose a physical unit of mass, and call it $m_{\rm code}$. As a unit of time we choose $Gm_{\rm code}/c^3$ and as a unit of length we choose $Gm_{\rm code}/c^2$. Since in GR we generally work in units where $G = c = 1$, this reduces the number of units from three to one, and mass, length and time can all be considered to 'have units of mass'.

In a simulation, all physical quantities are represented as dimensionless numbers in the computer. To add a physical interpretation, units must be assigned to these quantities when analysing the results. Since in a BBH system the only unit is that of mass, this reduces to choosing a mass scale $m_{\rm code}$, for the problem.

The initial data parameters for an equal mass, non-spinning binary black hole simulation can be specified as the sum of the horizon masses, m, an initial momentum p^i, and an initial radius r.

For example, for the R1 initial data set, the parameters are shown in Tab. I

Quantities derived from the simulation, for example the time of coalescence t_c, are also measured in the same mass unit $m_{\rm code}$. For example, we may obtain $t_c = 174\, m_{\rm code}$.

II. COMPARING TWO SIMULATIONS

Now suppose we have two simulations, A and B. Each has a mass scale, which we will label ${m_{\rm code}}^A$ and ${m_{\rm code}}^B$. The initial data parameters and simulation results for each simulation will be measured in their respective units.

Now consider the analysis of quantities derived from the simulations, for example t_c^A and t_c^B. The numbers that come out of a simulation will be measured according to the mass scale of that simulation, so for example, we might have $t_c^A = 174\, {m_{\rm code}}^A$ and $t_c^B = 160\, {m_{\rm code}}^B$. Now suppose that we have chosen $m^A = 1.01017\, {m_{\rm code}}^A$ and $m^B = 1.01017\, {m_{\rm code}}^B$, i.e. we have kept the numerical values of the total horizon masses of the two simulations the same. If we wish to add the intepretation that the two physical horizon masses in the simulations are the same, then we are asserting that $m^A = m^B$, which implies ${m_{\rm code}}^A = {m_{\rm code}}^B$. Therefore the two code units will be the same, and t_c^A and t_c^B can be compared directly; i.e. $t_c^A/t_c^B = 174\, {m_{\rm code}}^A/(160 {m_{\rm code}}^B) = 174/160$.

Quantity	Value
m	$1.01017\ m_{\rm code}$
p^i	$(0,\ 0.133,\ 0)\ m_{\rm code}$
r	$3.257\ m_{\rm code}$

TABLE I: Initial data parameters for R1

III. PHYSICAL INTERPRETATION

To add a physical intepretation to a simulation number, we must first re-insert G and c, and secondly set the mass scale $m_{\rm code}$. Inserting G and c gives the time of coalescence as $t_c = 174\, m_{\rm code} G/c^3$. To set the mass scale, we choose the horizon masses to be one solar mass (2×10^{30} kg) each. This gives $m/2 = 2 \times 10^{30}$ kg (where m is the total horizon mass given in the table above). Using this relation and that in the table above gives $m_{\rm code} = 3.96 \times 10^{30}$ kg. We have $G/c^2 = 7.41 \times 10^{-28}$ m and $G/c^3 = 2.47 \times 10^{-36}$ s. Finally, we can express the remaining parameters in physical units: $r = 3.257\, m_{\rm code} G/c^2 = 9.56$ km and $p^y = 0.133\, m_{\rm code} c = 1.58 \times 10^{38}\ {\rm kg\, m\, s^{-1}}$.